

In chapter 5 a method for obtaining a system in the primitive variables \mathbf{u} and p is presented. One starts from a semidiscretization in time of the Navier-Stokes equations by means of a time stepping method; then the continuity equation is eliminated to obtain a Poisson equation for the pressure. An integral condition for the pressure is then derived supplementing the system. Chapter 7 is devoted to a discussion of the fractional-step projection method for the primitive-variable Navier-Stokes equations. Chapter 8 is concerned with the incompressible Euler equations the emphasis being placed on discretizations by Taylor-Galerkin methods. Finally, a set of appendices provide expressions for vector differential operators in orthogonal curvilinear coordinates and other useful vector identities.

The book succeeds in achieving its intended goal. It gathers a wealth of useful information some of which is new while the rest is scattered in the literature. Although the mathematical background required is such that the book is accessible to students and beginning researchers, a significant amount of the material included can be only appreciated by the more experienced practitioner. There are however a great number of typos and minor grammatical offences which fortunately do not manage to destroy the otherwise flowing narrative.

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17[65-02, 65Lxx, 65Mxx]—*Theory and numerics of ordinary and partial differential equations*, by M. Ainsworth, J. Levesley, W. A. Light and M. Marletta, Oxford University Press, Oxford, 1995, xiii + 333 pp., 24 cm, \$62.00

This book, the fourth in the series *Advances in Numerical Analysis*, consists of lecture notes by six invited speakers at the SERC Summer School in Numerical Analysis that was held at the University of Leicester in July of 1994. The topics presented fall into two categories: ordinary and partial differential equations. The stated aim of the lectures is “to be accessible to beginning graduate students and to progress to a point where, by the final lectures, current research problems could be described”. In my opinion the lectures have succeeded admirably in this regard. Of particular note is the effort to make ancillary materials available to the reader. For example, Professor Corliss provides the reader with an anonymous ftp address for the LaTeX source file, MAPLE worksheets, and bibliography used in his lecture notes. Professor Johnson gives an anonymous ftp site and a WWW URL for various versions of the Femlab software package used in solving initial/boundary problems for ODE’s and PDE’s. Similarly, Professor Petzold provides electronic Internet references (via Netlib) to software for solving ODE’s and DAE’s.

In the area of ordinary differential equations the speakers and the title of their lectures are as follows:

1. George Corliss: *Guaranteed Error Bounds for Ordinary Differential Equations*, pp. 1–75.
2. Linda Petzold: *Numerical Solution of Differential-Algebraic Equations*, pp. 123–142.
3. Marino Zennaro: *Delay Differential Equations: Theory and Numerics*, pp. 291–333.

The lectures in partial differential equations are:

1. Claes Johnson (with Kenneth Eriksson, Don Estep, and Peter Hansbo): *Introduction to Computational Methods for Differential Equations*, pp. 77–122.
2. Ian Sloan: *Boundary Element Methods*, pp. 143–180.
3. Andrew Stuart: *Perturbation Theory for Infinite Dimensional Dynamical Systems*, pp. 181–290.

In Professor Corliss's lectures the author develops many of the ideas and techniques of interval analysis as they apply to initial-value problems for ODE's. He discusses the notion of a validated solution, the cost involved, and three techniques used in self-validating algorithms: (i) use of differential inequalities (ii) defect controlled solutions (iii) Lohner's method. Of these three methods the last is most practical and most general.

Professor Petzold provides a brief introduction to numerical methods, Runge-Kutta and multistep, for systems of the form $F(t, y, y') = 0$. For each of these classes of numerical method she provides a general convergence theorem and a discussion of the available software, most notably DASSL and its extensions.

In his presentation of DDE's, Professor Zennaro provides a nice introduction to the theory of delay differential equations as well as Runge-Kutta methods (theory and practice) for the solution of such problems. Considerable attention is paid to the stability properties of the methods introduced.

The adaptive numerical solution of differential equations using Galerkin methods with piecewise polynomial trial space, i.e. finite element methods, is the subject of Professor Johnson's article. As is pointed out in the section entitled Concluding remarks, the approach presented in these notes is quite different than the classical treatment of numerical methods for differential equations. The essential building blocks are a posteriori error estimates and, to a lesser extent, a priori error bounds. A particularly appealing aspect of this work is the basic uniform methodology for elliptic as well as time-dependent parabolic and hyperbolic PDE's. Much of this material is relatively new having been developed in the last several years.

Boundary element methods start with an integral (over the boundary) formulation of boundary value problems. By discretizing the boundary and defining corresponding piecewise polynomial spaces one obtains a discrete problem that is smaller than that given by the finite element method. The resulting matrix problem is dense and small as opposed to the FEM where one gets large and sparse matrices. In his lecture, Professor Sloan gives a concise survey of both Galerkin and collocation techniques as applied to boundary integral (single or double layer) equations.

The interpretation of data obtained from numerical procedures for initial value problems over *long intervals* of time is addressed in Professor Stuart's article. The framework for this study is in that of an abstract evolution equation acting in a Hilbert space. The main theme of the lecture is the study of the effect of perturbations on certain invariant (under the evolution equation) sets. This rather lengthy article is primarily theoretical in nature with remarks to applications such as the Cahn-Hilliard equation.

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